Indian J. Phys. **68A** (1), 95 – 104 (1994)

IJP A
— an international journal

A scalar-vector model of quark-antiquark interaction under linear confinement

S Chakrabarty *
International Centre for Theoretical Physics, Trieste, Italy

Received 29 March 1993, accepted 16 July 1993

Abstract : Considering the idea that the constituent quark mass is the dressed sum of current quark mass and dynamical quark mass, and using the standard values of current quark masses we obtain approximate values of constituent quark masses, which are then used in our extensively studied Bethe-Salpeter-reduced potential model. We find that the mass formulas become much simpler for linear potential ar with zero anomalous magnetic moment (λ), the values of 'a' in the linear potential being (1/5). Also, some of the quantities can be related to each other and the match with experimental data is good.

Keywords : Quantum chromodynamics, quarks, Bethe-Salpeter equation, quark mass.

PACS No. : 12.40.Qq

1. Introduction

The relativistic QCD suggests that there are three types of effective quark masses [1]. These are the dynamically generated quark mass $m_{\rm dyn}$ due to non-perturbative quantum chromodynamic cloud of gluons, the current quark masses related to current divergences and large momentum transfer events, and the constituent quark masses associated with hadrons. The constituent quark masses are given by [2,3], the dressed sum of dynamical and current quark masses. Elias *et al* [1] have shown that hadron masses can be understood on the basis of quark mass additivity by choosing the dynamical quark mass $m_{\rm dyn}$ as 320 MeV which lies within the theoretical value of $m_{\rm dyn}$ = 290–320 MeV generated through $\langle 0| : q(x)\ q(y) : |0\rangle$ [4-6].

* Permanent address : Department of Physics, Visva-Bharati University, Santiniketan-731 235, West Bengal, India.

Recently we have studied in detail the light and heavy mesons within a scalar-vector model of quark confinement where the spin-dependent force between quarks is given by the non-relativistic reduction of the Bethe-Salpeter equation into the Breit interaction [7-14]. In these work, we tried to understand especially the S and P wave meson spectra, the spin-orbit interaction and the quark masses. The quark masses were free parameters, Moreover, we gave only the results of comparative studies with different potentials, *e.g.* linear, harmonic and power-law.

In the present work, we consider the linear relation among the three types of quark masses in order to fix the constituent quark masses. We find that the quark masses obey simple empirical relations. Moreover, in this study we consider a linear potential model because it has met with much success. We fix some of the parameters like the quark anomalous magnetic moment (λ) and the quantity 'a' in the linear potential 'ar' on the basis of some physical arguments. Interestingly, one can obtain relations among different physical quantities because the mass formulas take simpler forms.

The plan of the paper is as follows. In Section 2 we discuss the quark mass. In Section 3 we describe the potential model. In Section 4 we give results and discussions. In Section 5 we give the concluding remarks.

2. The quark masses

We use the result that the constituent quark masses are dressed sum of dynamical and current quark masses [2,3] given by

$$m_{co} = m_{cu} \quad m_{dyn} . \tag{1}$$

The values of current quark masses are obtained from current algebra and QCD spectral sum rules (QSSR) in different hadronic channels which are found to give estimates of light and heavy current quark masses [15]. Assuming $m_u = m_d$ we have

$$m_u = 0.012 ; \quad m_c = 1.45 \quad \text{and} \quad m_b = 4.67 \ \text{(GeVs)} . \tag{2}$$

However, for s-quark we choose the current mass to be $m_s = 0.182$ GeV [16] because the prediction of Narison [15] on m_s seems to be very large. The addition of non-perturbative scale of 0.320 GeV gives the values for the constituent masses of u, s, c and b quarks as

$$m_u = 0.332 ; \ m_s = 0.502 ; \ m_c = 1.77 \ \text{and} \ m_b = 4.99 \ \text{(GeVs)}. \tag{3}$$

On the basis of the quark masses given in eq. (3), we can observe some empirical relations :

$$m_u = m_d = \frac{1}{3}\text{GeV}; \ m_s = \frac{3m_u}{2};$$

$$m_c = 5m_u; \ m_b = 15m_u . \tag{4}$$

Now, using eq. (4) the sum of quark masses are given by

$$m_u + m_s = 0.833,$$

$$m_u + m_c = 2,$$

$$m_u + m_b = 5.33 \text{ (GeVs)}. \tag{5}$$

These values compare well with the earlier predictions [12] which are 0.84, 2.02 and 5.36 GeVs respectively. Thus we find that the quark masses obtained by us are based on the relations between three types of quark masses suggested by QCD and are in conformity with the results of an earlier paper which shows some similarity as well as phenomenological connections with PCAC results cited in the work of Chakrabarty and Deoghuria [12]. Since the d, s, c, b quark masses are integral or half-integral multiples of u-quark mass, the whole picture becomes aesthetically more appealing. Accordingly, there are two types of matter— non-strange and strange, similar to two types of particles-bosons and fermions. Thus eq. (4) gives

$$m_u : m_d : m_s = 2 : 2 : 3,$$

$$m_s : m_c : m_b = 3 : 10 : 30, \tag{6}$$

$$m_u + m_d : m_u + m_s : m_u + m_c : m_u + m_b = 4 : 5 : 12 : 32, \tag{7}$$

$$2\left(m_s - m_u\right) = \frac{1}{4}\left(m_c - m_u\right) = \frac{1}{14}\left(m_b - m_u\right),$$

$$\frac{1}{7}\left(m_c - m_s\right) = \frac{1}{27}\left(m_b - m_s\right) = \frac{1}{20}\left(m_b - m_c\right). \tag{8}$$

Thus we find that the constituent quark masses as well as their differences obey approximate relations if they are obtained from eq. (1), using the known values of current and dynamical quark masses.

3. The BS reduced scalar-vector potential model

The potential model we are considering here is discussed in detail elsewhere [8-10]. We choose the potential as

$$V_0 = - k/r + V_v(r) + V_s(r) \tag{9}$$

where
$$V_v(r) = 0.25\, V_{\text{conf}}(r), \tag{10}$$

$$V_s(r) = 0.75\, V_{\text{conf}}(r), \tag{11}$$

and $K = (4/3)\,\alpha_s$. α_s is the strong interaction coupling constant.

Here

$$V_{\text{conf}}(r) = a\,r = \frac{r}{5} \qquad (\text{ in GeV unit }). \tag{12}$$

The spin-dependent part of the potential is [6]

$$V_{\text{spin}} = \frac{1}{m_1 m_2}\left[\lambda_1 F_1(r) \boldsymbol{L}.\boldsymbol{S} + \lambda_2 F_2(r) T_{12} + \lambda_3 F_3(r) \boldsymbol{S}_1.\boldsymbol{S}_2\right], \tag{13}$$

where m_1 and m_2 are the masses of the quarks and antiquarks respectively. We have $\lambda_1 = 3/2$, $\lambda_2 = 1/12$ and $\lambda_3 = 2/3$.

3.1. S-states of quarkonia

Using the mass formulae for a meson [9], the mass of the S-wave vector and pseudo-scalar mesons for linear potential with $\lambda = 0$, $a = 1/5$ and $\eta = 1/4$ are given by

$$M_v = m_1 + m_2 + C_1\mu^{-1/3} + \frac{1}{6m_1m_2}\left[\frac{16}{3}\pi\alpha_s|\psi(0)|^2 + \frac{C_2}{10}\mu^{1/3}\right] \tag{14}$$

and

$$M_p = m_1 + m_2 + C_1\mu^{-1/3} - \frac{1}{2m_1m_2}\left[\frac{16}{3}\pi\alpha_s|\psi(0)|^2 + \frac{C_2}{10}\mu^{1/3}\right]. \tag{15}$$

Using eqs. (14) and (15) we can calculate M_v, M_p and $M_v^2 - M_p^2$.

3.2. P-states of quarkonia

For a linear potential with $\lambda = 0$, $a = 1/5$ and $\eta = 1/4$ the radial functions take the forms††

$$F_1(r) = \frac{k}{r^3}, \tag{16}$$

$$F_2(r) = \frac{3k}{r^3} + \frac{1}{20r}, \tag{17}$$

$$F_3(r) = \frac{1}{10r} \tag{18}$$

Hence $F_1(r)$, $F_2(r)$ and $F_3(r)$ are related by

$$F_3(r) = 2\left[F_2(r) - 3F_1(r)\right]. \tag{19}$$

Also, the masses of the three ^3L_J levels M_J ($J = 0, 1, 2$) and the mass of the $^1L_{J=L}$ level M_1 are reduced to the forms [8]

$$M_0 = m' - \frac{4k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle, \tag{20}$$

$$M_1 = m' - \frac{k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{40m_Q^2}\left\langle\frac{1}{r}\right\rangle, \tag{21}$$

$$M_2 = m' + \frac{7k}{5m_Q^2}\left\langle\frac{1}{r^3}\right\rangle + \frac{3}{200m_Q^2}\left\langle\frac{1}{r}\right\rangle, \tag{22}$$

†† In eqs.(10), (11) and (12) of ref. [8], $r^{1/(2-\beta)}$ is incorrectly printed and should be read as $\frac{1}{r^{(2-\beta)}}$.

and $$M_1' = m' - \frac{1}{20m_Q^2}\left\langle\frac{1}{r}\right\rangle. \tag{23}$$

And $\overline{M}_1 - M_1'$, which is the difference between the centre of gravity of states and ${}^1P_1$ mass, is given by

$$\overline{M}_1 - M_1' = \frac{1}{15m_Q^2}\left\langle\frac{1}{r}\right\rangle. \tag{24}$$

The spin-orbit part of the confining interaction is given by [10]

$$M_{SO}^{CF} = -\frac{3}{40m_Q^2}\left\langle\frac{1}{r}\right\rangle = -\frac{9}{8}\left(\overline{M}_1 - M_1'\right). \tag{25}$$

We also obtain the expression for A_{11}, A_{12}, A_{13}, $\left\langle\frac{V'}{r}\right\rangle$, $\langle V''\rangle$ and $\left\langle\frac{S'}{r}\right\rangle$ as

$$A_{11} = \frac{k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle, \tag{26}$$

$$A_{12} = \frac{3k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{20m_Q^2}\left\langle\frac{1}{r}\right\rangle, \tag{27}$$

$$A_{13} = \frac{1}{10m_Q^2}\left\langle\frac{1}{r}\right\rangle, \tag{28}$$

$$\frac{1}{m_Q^2}\left\langle\frac{V'}{r}\right\rangle = \frac{k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle + \frac{1}{20m_Q^2}\left\langle\frac{1}{r}\right\rangle, \tag{29}$$

$$\frac{1}{m_Q^2}\langle V''\rangle = -\frac{2k}{m_Q^2}\left\langle\frac{1}{r^3}\right\rangle, \tag{30}$$

$$\frac{1}{m_Q^2}\left\langle\frac{S'}{r}\right\rangle = \frac{3}{20m_Q^2}\left\langle\frac{1}{r}\right\rangle. \tag{31}$$

Thus, we find that A_{11}, A_{12} and A_{13} obey a relation similar to that satisfied by F_1, F_2 and F_3 (cf. eq. 19) and is given by

$$A_{13} = 2\left(A_{12} - 3A_{11}\right). \tag{32}$$

We also find that $\left\langle\frac{S'}{r}\right\rangle$, $\left\langle\frac{V'}{r}\right\rangle$ and $\langle V''\rangle$ are related by

$$2\langle S'/r\rangle = 3[\langle V'/r\rangle + \langle V''\rangle. \tag{33}$$

Now, using the expression for $\left\langle\frac{1}{r^3}\right\rangle$ and $\left\langle\frac{1}{r}\right\rangle$ given by the scaling relation [10] we can write eqs. (20) – (31) in the following forms :

$$M_0 = m' - \frac{2k\,C_3}{m_Q}, \tag{34}$$

$$M_1 = m' - \frac{kC_3}{2m_Q} + \frac{C_2}{40(2)^{1/3}}\, m_Q^{-5/3}, \tag{35}$$

$$M_2 = m' + \frac{7k\,C_3}{10m_Q} + \frac{3C_2}{200(2)^{1/3}}\, m_Q^{-5/3}, \tag{36}$$

$$M_1' = m' - \frac{C_2}{20(2)^{1/3}}\, m_Q^{-5/3}, \tag{37}$$

$$\overline{M}_1 - M_1' = \frac{C_2}{15(2)^{1/3}}\, m_Q^{-5/3} = -\frac{8}{9}\, M_{SO}^{CF}, \tag{38}$$

$$A_{11} = \frac{kC_3}{2m_Q}, \tag{39}$$

$$A_{12} = \frac{3k\,C_3}{2m_Q} + \frac{C_2}{20(2)^{1/3}}\, m_Q^{-5/3}, \tag{40}$$

$$A_{13} = \frac{C_2}{10(2)^{1/3}}\, m_Q^{-5/3}, \tag{41}$$

$$\frac{1}{m_Q^2}\left\langle \frac{V'}{r} \right\rangle = \frac{k\,C_3}{2m_Q} + \frac{C_2}{20(2)^{1/3}}\, m_Q^{-5/3}, \tag{42}$$

$$\frac{1}{m_Q^2}\langle V'' \rangle = -\frac{k\,C_3}{m_Q}, \tag{43}$$

$$\frac{1}{m_Q^2}\left\langle \frac{S'}{r} \right\rangle = \frac{3C_2}{20(2)^{1/3}}\, m_Q^{-5/3} \tag{44}$$

Interestingly, we have simple relationships among different quantities for $c\bar{c}$ and $b\bar{b}$, which are :

$$\left(A_{11}\right)_{b\bar{b}} = \frac{1}{3}\left(A_{11}\right)_{c\bar{c}}, \tag{45}$$

$$\frac{1}{m_b^2}\langle V'' \rangle_{b\bar{b}} = \frac{1}{3m_c^2}\langle V'' \rangle_{c\bar{c}}, \tag{46}$$

also we can see that $\left(\overline{M}_1 - M_1'\right)_{c\bar{c}}$ and $\left(\overline{M}_1 - M_1'\right)_{b\bar{b}}$ are now connected by a simple relation obtained from eqs.(4) and (24) as

$$\frac{\left(\overline{M}_1 - M_1'\right)_{c\bar{c}}}{\left(\overline{M}_1 - M_1'\right)_{b\bar{b}}} = 3^{5/3} \tag{47}$$

Considering $(\overline{M}_1 - M_1')_{b\bar{b}} = 5.4$ MeV. We get from eq.(47) $(\overline{M}_1 - M_1')_{c\bar{c}} = 33.67$ MeV. Next considering eq. (25) for M_{SO}^{CF} in a similar way, we have

$$\left(M_{SO}^{CF}\right)_{c\bar{c}} \left(\overline{M}_1 - M_1'\right)_{b\bar{b}} = \left(\overline{M}_1 - M_1'\right)_{c\bar{c}} \left(M_{SO}^{CF}\right)_{b\bar{b}} . \tag{48}$$

For $\left(M_{SO}^{CF}\right)_{b\bar{b}} = -7.7$ MeV given by experimental data, eqs. (47) and (48) give $\left(M_{SO}^{CF}\right)_{c\bar{c}} =$ 48.01 MeV. It will be easy to apply this model to the study of decays [13,14].

4. Results and discussion

The quantities C_1, C_2, b for non-self conjugate mesons, C_1, C_2 for self-conjugate mesons or C_1, C_2, C_3 for P-states $c\bar{c}$ and $b\bar{b}$ are obtained from input masses of mesons as in the work of Deoghuria and Chakrabarty [10]. In earlier work, there were 17 parameters (η, β, Λ, λ, ; C_1, C_2, b ; C_1, C_2 ; C_1, C_2, C_3 ; m_u, m_d, m_s, m_c, m_b) whereas in the present work, we have 11 parameters (η, Λ, C_1, C_2, b ; C_1, C_2 ; C_1, C_2, C_3 ; m_u). The leptonic widths for ψ and γ have been taken to be input for calculating $|\Psi(0)|^2_{c\bar{c}}$ and $|\Psi(0)|^2_{b\bar{b}}$. The QCD scale parameter (Λ) and scalar-vector fraction (η) have been taken to be free parameters. The quark masses and other parameters are given in Table 1. For linear potential, the choice of $a = (1/5)$ GeV^2

Table 1. Parameters used in Table 2 and 3 are given here.

Parameters	Quark masses (GeVs)	Input leptonic widths
$V = ar$	$m_u = \frac{1}{3}$	Leptonic width of ψ = 4.6 KeV
$a = 1/5$ (GeV unit)	$m_d = \frac{1}{3}$	Leptonic width of γ = 1 22 KeV
$\beta = 1$	$m_s = \frac{1}{2}$	
$\lambda = 0$		
$\Lambda = 0.100$ GeV	$m_c = \frac{5}{3}$	
$\eta = 0.25$	$m_b = 5$	
	$m_t = 60$	
Non-self conjugate mesons :		
$C_1 = -0.01856$	$C_2 = -2.12419$	$b = 0.95388$
Self-conjugate mesons (S state) :		
$C_1 = -0.249218$; $C_2 = 2.812694$		
1P-states of self-conjugate mesons :		
$C_1 = 0.164504$	$C_2 = 3.05416$	$C_3 = 0.271033$

follows from the string model of hadron [17,18]. The quark anomalous magnetic moment λ is taken to be zero because there is no evidence for a non-zero value. Moreover Weinberg has argued in a recent work that quarks should have no anomalous magnetic moment [19]. The calculated values of masses of vector (M_v) and pseudoscalar (M_p) mesons and $M_v^2 - M_p^2$ are given in Table 2. We find that the overall match the data is good. However, the results is bad for light mesons which is not very unlikely because this model should work best for heavy

Table 2. Predicted masses of vector (M_V), pseudoscalar (M_p) mesons and hyperfine splittings ($M_V^2 - M_p^2$) along with experimental data are given here.

Hyperfine Meson Symbols (M_V, M_p)	M_V (MeV)	Experimental data (MeV)	M_p (MeV)	Experimental data (MeV)	$M_V^2 - M_p^2$ GeV^2	Experimental data (MeV)
(ρ, π)	751.66	769	276.76	139	.488	0.573
(K^{*+}, K)	892.00	892	530 36	494	.514	0.551
(ϕ, η_s)	1042.85	1020	753.58		.520	
($D^{*\dagger}$, $D^\dagger$)	2007.20	2007.2 ± 2 1	1864 60	1864.6 ± 0.6	.552	0.552
(F^*, F)	2173.05	2113 ± 8	2045.42	1970.5 ± 2.5	.538	0.569 ± 0.06
(B^*, B)	5319.17		5266.39	5277 9 ± 1.1	.559	
(B_s^*, B_s)	5488.28		5438.58		543	
(B_c^*, B_c)	6660.26		6616.96		.575	
(T_u^*, T_u)	60307.67		60303.03		.560	
(T_s^*, T_s)	60477.67		60473.18		.544	
(T_c^*, T_c)	61652.01		61647 43		.565	
(T_b^*, T_b)	64989.94		64985.58		.567	
($\psi^\dagger$, $\eta_c^\dagger$)	3097.00	3097	2983.00	2983	.693	0.693
(Υ, η_b)	9829.20	9460	9777.90		1.006	

† Input

quarks. The results for P states are given in Table 3. The quantities $M({}^1p_1)$, $\overline{M}_1 - M_1'$ and M_{SO}^{CF} match with the data quite well. Further A_{11}, A_{12}, A_{13}, $\left\langle \frac{V'}{r} \right\rangle$, $-\langle V'' \rangle$ and $\left\langle \frac{S'}{r} \right\rangle$ also show correct behaviour [8].

5. Conclusions

Thus we find that we have suggested a simpler potential model in which the parameters are less in number compared to our earlier work [10]. We find that the model gives relations among various quantities and can explain the mass spectra of S and P states.

Table 3. The predicted masses of $^1P_1$ states, $\overline{M}_1 - M_1'$ values and M_{SO}^{CF} for $c\bar{c}$ and $b\bar{b}$ as well as numerical values of expectation values of different quantities required for calculating the spin splitting using the known masses of three ^3L_j level of $c\bar{c}$ as input for parameters given in Table 1. Input masses for $c\bar{c}$ are given in ref. [8].

	$c\bar{c}$		$b\bar{b}$	
	Predicted value (MeV)	Experimental value (MeV)	Predicted value (MeV)	Experimental value (MeV)
$M(^1P_1)$	3456.41	3525.4 ± 0.8	10112.92	9894.8 ± 1.5
$\overline{M}_1 - M_1'$	68.98	0±	11.05	5.4 ± 1.5
M_{SO}^{CF}	-77.60	-25.9 ± 1 1	-12.44	-7.7 ± 2.1
A_{11}	23.311		6.431	
A_{12}	121.667		27.584	
A_{13}	103.467		16.580	
$\left\langle \frac{V'}{r} \right\rangle$	75.044		14.721	
$\langle V'' \rangle$	-46.622		-12.862	
$\left\langle \frac{S'}{r} \right\rangle$	155.2		24.871	

Acknowledgments

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank Professors Q Shafi, F Hussain and A O Barut for encouragement and S Deoghuria for his assistance in computation.

References

[1] V Elias, T Mong and M D Scadron 1989 *Phys. Rev.* **D40** 3670

[2] A W Henry and D B Lichtenberg 1985 *Fortschr. Phys.* **33** 139

[3] D B Lichtenberg 1989 *Phys. Rev.* **D40** 3675

[4] V Elias, T G Stelle and M D Scadron 1988 *Phys. Rev.* **D38** 1584

[5] V Elias and M D Scadron 1984 *Phys. Rev.* **D30** 647

[6] L J Reinders and K Stam 1986 *Phys. Letts.* **180B** 125

[7] S Chakrabarty 1989 *J. Phys. Mal.* **10** 99

[8] S Deoghuria and S Chakrabarty 1989 *J. Phys.* **G15** 1213

[9] S Chakrabarty and S Deoghuria 1990 *J. Phys.* **G16** 185

[10] S Deoghuria and S Chakrabarty 1990 *J. Phys.* **G16** 1825

[11] S Chakrabarty 1991 *Fortschr. Phys.* **39** 139

[12] S Chakrabarty and S Deoghuria 1992 *J. Phys.* **G18** 739

[13] S Deoghuria and S Chakrabarty 1992 *Z. Phys.* **C53** 293

[14] S Deoghuria and S Chakrabarty 1992 *Z. Phys.* **C55** 687

[15] S Narison 1988 *Light and heavy quark masses, test of PCAC and flavour breakings of condensates in QCD. ICTP* Preprint IC/88/171 · PM/88/35

[16] S Weinberg 1977 *A. Festschrift for I.I.* Rabi ed. L. Motz (New York New York Academy of Sciences) p 185

[17] Y Nambu 1974 *Phys. Rev.* **D10** 4262

[18] T P Cheng and L F Li 1984 *Gauge Theory of Elementary Particle Physics* (Oxford)

[19] S Weinberg 1990 *Phys. Rev. Lett.* **65** 1181